Electrostatics:

Branch of Physics which deals with the study of electric forces, fields, and potentials arising charges at rest.

Introduction:

- In 600 B.C., the Greek Philosopher Thales observed that amber, when rubbed with wool, acquires the property of attracting objects such as small bits of paper, dry leaves, dust particles, etc.
- This kind of electricity developed on objects, when they are rubbed with each other, is called frictional electricity.

Elecric Charges:-

Electric charge is a fundamental property associated with elementary particles for example electron, proton and which responsible for electric forces , field and potential

Kinds Of Charges:-Two kinds

(i) negative charge (ii) positive charge

Cause Of Charging: Transfer of electron.
Material bodies consist of large number of electrons and protons in equal number and hence are in neutral in their normal state. But when the body is rubbed for example when a glass rod is rubbed with silk cloth, electrons are transferred from glass rod to silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged as it recieves extra electrons from the glass rod.

- **A** body acquire positive charge due to loss of electron and acquire negative charge due to gain of electron.

Or defficiet of electron makes abody positive while excess of electrons makes a body negative.

Polarity:

it is properties of charges which differentiates the kind of charges.

- ✓ Electrons are negative and protons are positivecharge particles. And neutrons are uncharged particles.
- The American scientist Benjamin Franklin introduced the concept of positive and negative charges
- In the table given, if an object in the first column is rubbed against the object given in second column, then the object in the first column will acquire positive charge while that in second column will acquire negative charge.

Positive	Negative
Woollen cloth	Rubber shoes
Woollen cloth	Amber
Woollen cloth	Plastic object
Fur	Ebonite rod
Glass rod	Silk cloth

Series of material when rubbed with each other the former of series form positive charge and later forms negative charge.

1. Fur 2. Flannel 3.Sealing wax 4.Glass 5.Cotton 6.Paper 7.Silk 8.Human body 9.Wood 10.Metal 11.Rubber 12.Resin 13. Amber 14. Sulphur 15. Ebonite 16. Guta parcha

- ✓ Electric charge is a scalar quantity.
- ✓ It has unit- coulomb(C)
- ✓ Charge of an electron e =- 1.6×10^{-19} C
- ✓ Charge of a proton =-+1.6×10^{-19} C
- Charge of eletron is the smallest charge exist independently in nature , so it is known as basic charge.
- ✓ Electron and proton composed of quarks having charges $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$. Quark does not exist in free state.

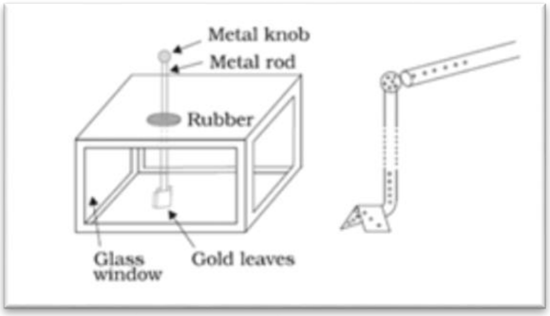
Law of charges:

Like charges repel each other whereas unlike charges attract each other.

ELECRTOSCOPE (GLE)

A simple apparatus used to detect charge on a body is the gold-leaf electroscope.

It consists of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge.



- ✓ The degree of divergance is an indicator of the amount of charge.
- ✓ Divergences increases when like charges brought in contact with metal knob.

Conductors:*The substances which allow electricity to pass through them easily are called conductors.*

Example – All the metals are good conductors.

- ✓ Conductors have electrons that can move freely inside the material.

Insulators:*The substances which do not allow electricity to pass through them easily are called insulators.*

- ✓ Most of the non-metals such as porcelain, wood, nylon, etc. are examples of insulator.
- ✓ If some charge is put on an insulator, then it stays at the same place.

Earthing:- When we bring a charged body in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body). This process of sharing the charges with the earth is called *grounding or earthing*.

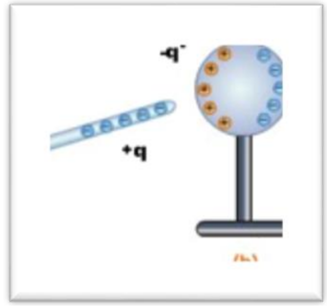
- ✓ A thick metal plate is buried deep into the earth and thick wires are drawn from this plate; these are used in buildings for the purpose of earthing near the mains supply.
- ✓ Earthing provides a safety measure for electrical circuits and appliances.

Methods of charging:

(i) Induction (ii) Conduction (iii) Friction

Induction:

When an insulator brought near a charged body but without contact unlike charges are developed at near face and like charges are developed at far face of insulated object.



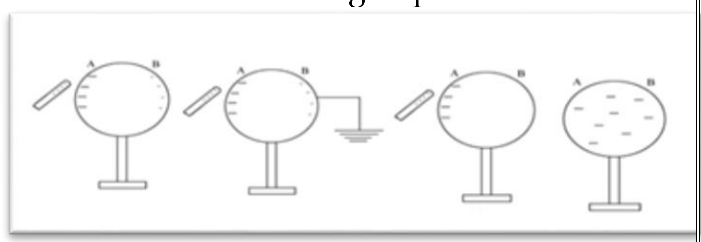
The amount of charged induced insulating body is $q' = -q\left(1 - \frac{1}{K}\right)$

Where q′ is known as induced charge and q is inducing charge , K is dielectric constant of insulting body.

Charging By Induction:

Do Obtain A Negatively Charged Body

A conductor may be charged permanently by induction in the following steps.



Step I

Bring a positively charged glass rod close to an conductor AB plaed over an insulating stand. The end A of the conductor becomes negatively while the far end B becomes positively charged. It is so because glass rod attracts the free electrons present in the conductor towards it. As a result, the electron accumulates at the near end A and end B becomes deficient of electrons and acquires positive charge.

Step II

The conductor is now connected to the earth. The positive charges will disappear. The negative induced charge on end A remains bound to it due to the attractive forces exerted by the positive glass rod.

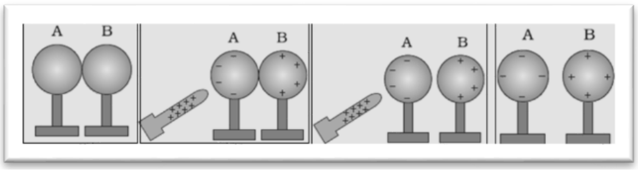
Step III

The conductor is disconnected from the earth keeping the glass rod still in its position. End A of the conductor continues to hold the negative induced charge.

Step IV

Finally, when the glass rod is removed, the negative induced charge on the near end spreads uniformly over the whole conductor.

To Obtain Two Opposite Charged Bodies



- Bring two metal spheres, A and B, supported on insulating stands, in contact
- Bring a positively charged rod near one of the spheres, say A, taking care that it does not touch the sphere.

The free electrons in the spheres are attracted towards the rod. This leaves an excess of positive charge on the rear surface of sphere B.

- Separate the spheres by a small distance while the glass rod is still held near sphere A .
- Remove the rod. The charges on spheres rearrange themselves as shown in Fig.

Charging By Friction Or Frictional Electricity

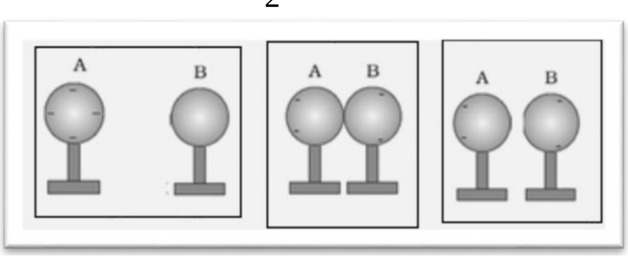
The process of electrification or charging when two insulationg bodies rubbed with each other.

- If we pass a comb through hairs, comb becomes electrically charged and can attract small pieces of paper.
- Mass of particle inreases when a particle acquire negative charge.

Charging By Conduction

When a charged body brought in contanct with a neutral object , then the charge is shared between the two bodies and the neutral object , acquire same charge that of charged body .

✓ If q_1 and q_2 are the charges present in two bodies A and B then the charge acquired each body when they are sepated after being contact is $\frac{q_1+q_2}{2}$



Basic Properties Of Electric Charges

(i)Additivity Of Charges:

The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on the different parts of the object.

✓ If $q_1, q_2, q_3, \ldots$ are electric charges present on different parts of an object, then total electric charge on the object, $q = q_1 + q_2 + q_3 + \ldots$

(ii) Charge Is Conserved

The total charge of an isolated system is always conserved.

✓ ***Charge cannot be created or destroyed but charged particles can be created or destroyed due to transfer of electrons.***

- ❖ Ex: when a neutal glass rod rubbed with a neutral silk , the glass rod acquire positive charge and silk acquire negative charge, but both acquire same magnitude of charges. Therefore net charge after rubbing also zero.
- ❖ Annihilation :$e^+ + e^- = \gamma$

charge before and after annihilation is zero

- ❖ Pair Production :$\gamma = e^+ + e^-$
- ❖ Nuclear decay

$${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

- ❖ Chemical reaction.

(iii) Quantization Of Charge

✓ ***The total charge of a body is not contnious value but always in discrete values.***

✓ ***The total charge of a body is always some integral multiple of elementary charge, e (= ± 1.6 × 10^{-19} C). This is known as quantization of charge.***

✓ Thus charge q on a body is always denoted by q = ne ,where n = any integer positive or negative

Q. ***Why quntisation is neglected in macroscopic charges?***

Ans: At the macroscopic level, the value of charge is very large as compared to the magnitude of charge *e*. Since $e = 1.6 \times 10^{-19}$ C

Ex.: A charge of magnitude, say 1 μC, contains something like 10^{13}times the electronic charge. So charge of some electrons in this scale also can increase or decrease negligible. Thus, at the macroscopic level, the quantisation of charge has no practical consequence and can be ignored.

Coulomb's Law

Two point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between themand acting along the line joining the two charges.

$F \alpha q_1 q_2$ *and* $F \alpha \dfrac{1}{r^2}$

$F = k\dfrac{q_1q_2}{r^2}$, Where, $k = \dfrac{1}{4\pi\varepsilon_0}$ [In SI, when the two charges are located in vacuum]

Now, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,Nm^2C^{-2}$$,

ε_0-Absolute permittivity of free space

= 8.854 × 10^{-12} C^2 N^{-1} m^{-2}

We can write $F_{vac} = 9\times10^9\dfrac{q_1q_2}{r^2}$

✓ These forces are attractive for unlike charges and repulsive for like charges.

Coulomb's Law in a A Medium

The force between two harges q_1 and q_2 located at a distance r in a medium may be expressed as

$$F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$$

✓ $\varepsilon = K\varepsilon_0$, K is known as dielectric constant of a medium ,Where ε-Absolute permittivity of the medium

$$F_{med} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{r^2}$$

✓ The ratio $\dfrac{\varepsilon}{\varepsilon_0}$ is denoted by ε_r, which is called ***relative permittivity*** of the medium with respect to vacuum.

✓ It is also called dielectric constant of the medium.

Relation between force in vacuum and medium:

$$\frac{F_{vac}}{F_{med}} = K$$

$$F_{med} < F_{vac} \because K > 1$$

$$F_{med} = \frac{F_{vac}}{K}$$

$K = \infty$ for conductor and K=80 for water.

K is unitless and dimensionless quantity.

✓ K=1 for CGS system in air or vacuum medium, Coulomb's force in CGS system is

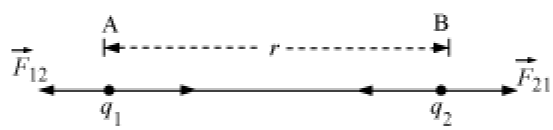
$$F_{vac} = \frac{q_1q_2}{r^2}$$

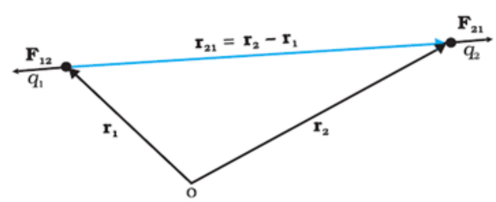
Dielectric Constant Or Relative Permittivity :

Dielectric Constant or Relative Permittivity of a medium is defined as the ratio of force between two charges separated by certain distance in vacuum and force between same two charges separated by same distance of separation in that of medium.

Coulomb's law in vector form

case:- 1:-for like charges $(q_1q_2 > 0)$





Consider two likecharges q_1 and q_2 present at points A and B in vacuum at a distance r apart. According to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on harge q_2 due to q_1) is given by,

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

Let $\hat{r}_{21}$ − unit vector pointing from charge q_1 to q_2

$\hat{r}_{12}$ − unit vector pointing from charge q_2 to q_1

So, $\hat{r}_{12} = -\hat{r}_{21}$

$\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\hat{r}_{12}$ [$\because$ $\vec{F}_{12}$ Is along the direction of unit vector $\hat{r}_{12}$] …(ii)

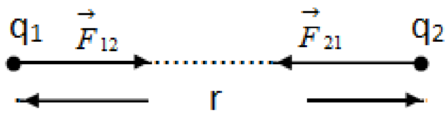
$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$ [$\because$ $\vec{F}_{21}$ is along the direction of unit vector $\hat{r}_{21}$] …(iii)

As $\hat{r}_{12} = -\hat{r}_{21}$, ∴Equation (ii) becomes

$\vec{F}_{12} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$---------------(iv)

On comparing equation (iii) with equation (iv), we obtain $\vec{F}_{21} = -\vec{F}_{12}$

Case-2 For Unlike Charges $(q_1 q_2 < 0)$



We have , $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$ and

$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$

$\therefore \vec{F}_{21} = -\vec{F}_{12}$

$\vec{F}_{21} + \vec{F}_{12} = 0$

Significance of vector form:

It states Coulomb's law obeys Newton's third law.

It is a central force.

$F_{12} : F_{12} = 1:1$

Definition of coulomb: One oulomb is that charge which exert a force of 9×10^9 N to aother charge of same magnitude , when separted at a distance of oe meter in air or vacuum.

Units of charge:

SI Unit: coulomb(C)

CGS Units:

(I) electostaitcs unint of charge (e.s.u of charge) or stat coulomb:

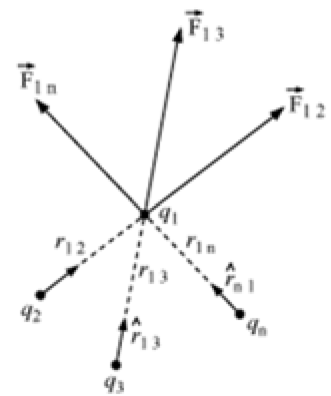
1C= 3×10^9stat-coulomb or frankline(Fr)

(II)electromagnetic unit of charge (e.m.u of charge) or ab coulomb.

1C=1/10 abcoulomb

Forces Between Multiple Charges Or Principle Of Superposition :

Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.



Consider that n point charges $q_1, q_2, q_3, \dots q_n$ are distributed in space in a discrete manner. The charges are interacting with each other.

Let the charges $q_2, q_3, \dots q_n$ exert forces $\vec{F}_{12}$, $\vec{F}_{13}$, ……., $\vec{F}_{1n}$ on charge q_1.

Then, according to principle of superposition, the total force on charge q_1 is given by,

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ... + \vec{F}_{1n}$

If the distance between the charges q_1 and q_2 is denoted as r_{12} and $\hat{r}_{12}$ is unit vector from charge q_2 to q_1, then, $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$

Similarly, the force on charge q_1 due to other charges is given by,

$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$ and $\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}$

Substituting these in equation (i),

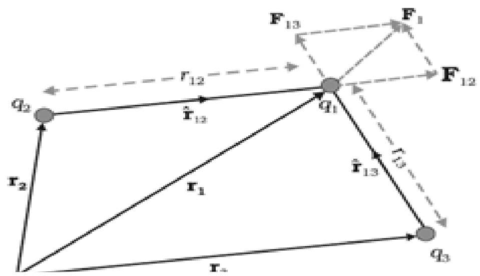
$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_2}{r_{13}^2}\hat{r}_{13} + ... + \frac{q_1 q_2}{r_{1n}^2}\hat{r}_{1n}\right)$$

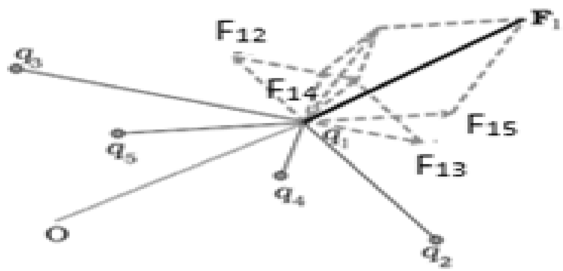
$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_1 q_i}{r_{1i}^2}\hat{r}_{1i}$$

Graphically:-

The vector sum is obtained by parellogram law of addition of vector.

✓ Similarly force on any other charge due to remaining charges say on q_2, q_3 etc. can be found by adopting this method.





Electric Field :

It is the space around a charge, in which any other charge experiences an electrostatic force.

- ✓ A charge is a soure of an electric field.
- ✓ **Test charge(q_0)**:It is a small vanishingly positive charge , used to detect the existence of source of electric field. It taken vanishingly small charge so that the electric field due to source charge must not affected.

Electric Field Intensity(E):

The electric field intensity at a point due to a source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$, where, $\vec{E}$ - Electric field intensityand $\vec{F}$ -Force experienced by the test charge q_0.

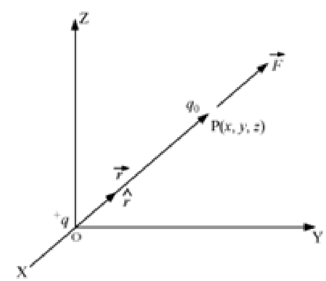
- ➢ $\vec{F} = q_0 \vec{E}$ or in magnitude , F = q_0E
- ➢ Electric field is a vector quantity and since $\vec{F} = q_0 \vec{E}$ and the direction of E is along the direction of F.
- ➢ $\vec{E}(\vec{r})$ is position dependended.
- ➢ The direction of electric field is along the dirction which a positive test charge experience force.
- ➢ It is radially outwards due to a positive source charge and radialy inward due to a source of negative charge.
- ➢ The strength of electric field is same at all point of spherical surface whose centre contains a point charge so electric field is spherical symmetry in nature.
- ➢ Unit of electric field is (N C^{-1})or Vm^{-1}

<u>Electric Field Due To A Point Charge</u>

We have to find electric field at point P due to point charge +q placed at the origin such that $\vec{op} = \vec{r}$

To find the same, place a vanishingly small positive test charge q_0 at point P.

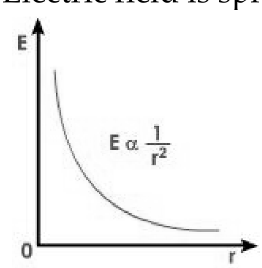
According to Coulomb's law, force on the test charge q_0 due to charge q is $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$



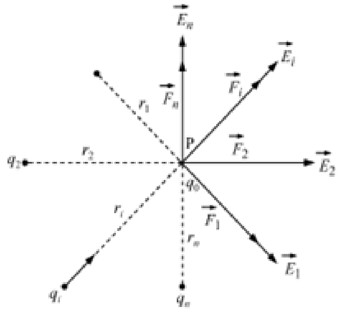
If $\vec{E}$ is the electric field at point P, then

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

- ✓ The magnitude of the electric field at point P is given by, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
- ✓ Electric field is spherical symmetric



Electric Field Due To A System Of Charges



Consider that n point charges q_1, q_2, q_3, … q_n exert forces $\vec{F}_{12}$, $\vec{F}_{13}$, …., $\vec{F}_{1n}$ on the test charge placed at origin O.

Let $\vec{F}i$ be force due to i^{th} charge q_i on q_0. Then,

$$\vec{F}i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{r}_i$$

Where, r_i is the distance of the test charge q_0 from q_i

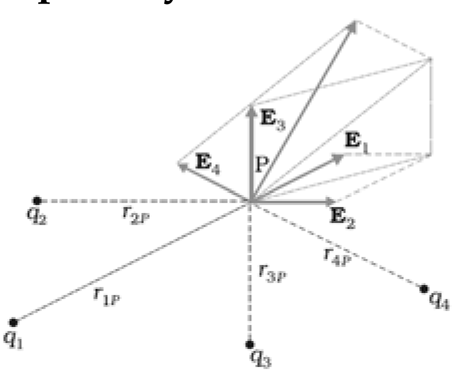
The electric field at the observation point P is given by, $\vec{E}_i = \lim_{q_0\to 0}\frac{\vec{F}_i}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^2}\hat{r}_i$

If $\vec{E}$ is the electric field at point P due to the system of charges, then by principal of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ...+\vec{E}_n = \sum_{i=1}^{n}\vec{E}_i$$

Using equation (i), we obtain $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$

Graphically:-

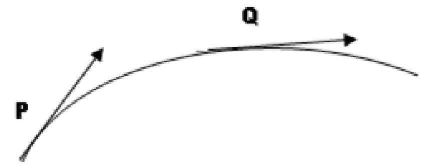


Electric Field Lines:

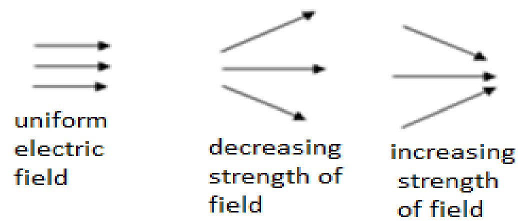
An electric line of force is the path along which a unit positive charge would move, if it is free to do so.

Properties Of Electric Lines Of Force

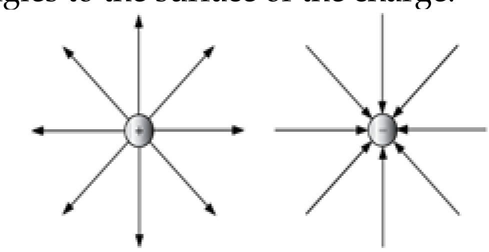
- ✓ An electric field line is, a curve ,such a way that the tangent drawn to it at any point gives the direction of net field at that point.



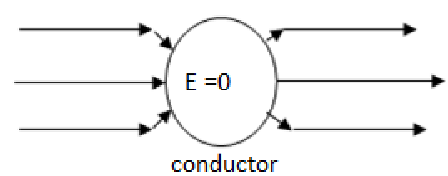
- ✓ The magnitude of the field is indicated by the density of the field lines.i.e, Magnitude is strong near the center where the field lines are close together, and weak , where they are relatively far apart.



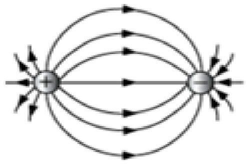
- ✓ These start from the positive charge and end at the negative charge.
- ✓ They always originate or terminate at right angles to the surface of the charge.



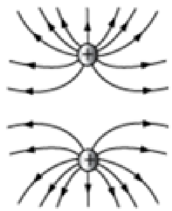
- ✓ They can never intersect each other because it will mean that at that particular point, electric field has two directions. It is not possible.
- ✓ They do not pass through a conductor. this shows that electric field inside a conductor is always zero.



- ✓ They contract longitudinally signifies thal unlike charges attract eachother.



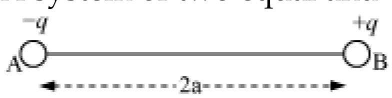
- ✓ They exert a lateral pressure on each othersignifies that like charges repel each other.



- ✓ Electric field lines are continuous curves in a charge free region.
- ✓ Electric fieldlines don't form closed loop.

Electric Dipole

A system of two equal and opposite charges



Electric Dipole Moment :–

It is a vector quantity, with magnitude equal to the product of either of the charges and the length of the electric dipole $\vec{P} = q(2\vec{a})$

- ✓ Its direction is from the negative charge to the positive charge.

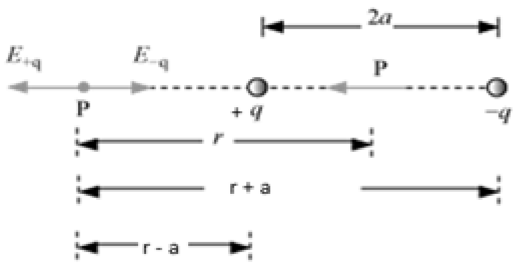
Ideal or Short or Point Dipole

The dipole in which the dipole length is infinitesimally small and the magnitude of charge is infinitely large such that the it has a finite dipole moment.

i.e, $\pm q \to \infty$ and $2a \to 0$

Electric Field On Axial Line /End-On-Position / Tan-A-Position Of An Electric Dipole

Consider a dipole of charges –q and +q and dipole length 2a.



Let P be at distance r from the centre of the dipole on the side of charge q. Then,

$$\vec{E}_{-q} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

Where, $\hat{p}$ is the unit vector of dipole moment $\vec{p}$ and acting along the dipole axis (from $-q$ to q).

Also, $\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p}$

The total field at P is $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right)\hat{p}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{4ar}{[(r-a)(r+a)]^2}\right)\hat{p}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2r\,q(2a)}{(r^2-a^2)^2}\right)\hat{p} \qquad \because p = q(2a)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2rp}{(r^2-a^2)^2}\right)\hat{p}$$

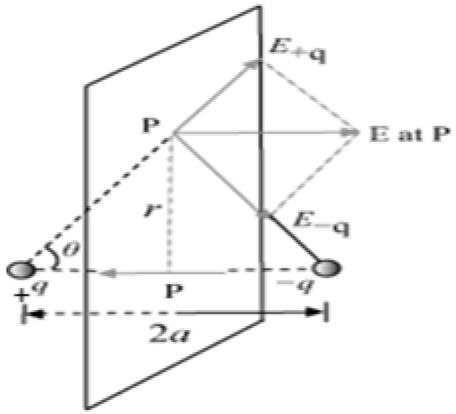
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2r\vec{p}}{(r^2-a^2)^2}\right)$$

Special case:-(*for a short or ideal or point dipole,r >> a)* $\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$ The direction of electric field is along the direction of dipole moment.

Electric Field For Points On The Equatorial Plane /Broard-Side –On Position / Tan-B Positon

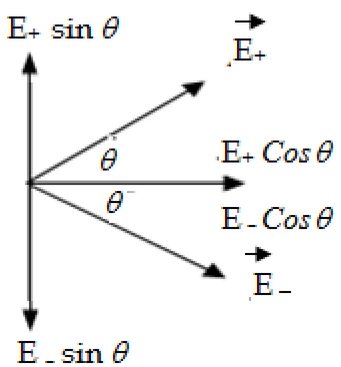
The magnitudes of the electric field due to the two charges $+q$ and $-q$ are given by,

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2} \quad \text{and} \quad E_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2}$$



Therefore $E_{+q}=E_{-q}$

The directions of E_{+q} and E_{-q} are as shown in the figure.



If we resolve E_{+q} and E_{-q} each into two perpendicular compents as in figure . The components normal to the dipole axis cancel away. The components along the dipole axis add up.

∴ Total electric field

$\vec{E} = -(E_{+q} + E_{-q})Cos\theta\,\hat{p}$, [Negative sign shows that field is opposite to $\hat{p}$]

$$\vec{E} = -2E_{+q}Cos\theta\,\hat{p} \qquad \because E_{+q} = E_{-q}$$

$$\vec{E} = -2\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2}\right)\left(\frac{a}{\sqrt{r^2+a^2}}\right)\hat{p}$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q(2a)}{(r^2+a^2)^{3/2}}\hat{p}$$

At large distances ($r >> a$), this reduces to

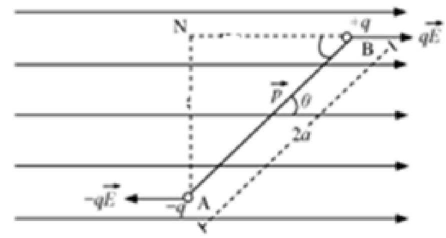
$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q(2a)}{r^3}\hat{p}$$

$$\boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}} \text{..........}where,\ p = q(2a)$$

The direction of electric field in opposite directin of dipole moment . $\frac{E_{axial}}{E_{equirial}} = \frac{2}{1}$

Dipole In A Uniform External Field

Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$placed in a uniform electric field $\vec{E}$ making an angle θ with electric field.



Force on charge $-q$ at A, $= -q\vec{E}$ (opposite to $\vec{E}$)

Force on charge $+q$ at B, $= q\vec{E}$ (along $\vec{E}$)

Net force on dipole $= q\vec{E} + (-q\vec{E}) = 0$

Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.

τ = Force × Perpendicular distance between the two forces

$\tau = qE\,(AN) = qE\,(2a\sin\theta)$

$\tau = q(2a)\,E\sin\theta$

$\tau = pE\sin\theta$

In vector form: $\vec{\tau} = \vec{p}\times\vec{E}$

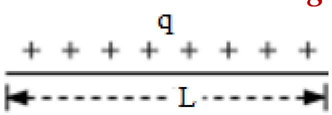
- ✓ A electric dipole experience no net force but experience net torque in a uniform electric field.
- ✓ Maximum torque $\tau_{max} = pE$ *when* $\theta = 90^0$
- ✓ $\tau_{min} = 0$ *when* $\theta = 0^0$or 180^0
- ✓ $\theta = 0^0$ (dipole is said to be stable equilibrium)
- ✓ $\theta = 180^0$ (dipole is said to be unstable equilibrium)

Continuous Charge Distribution:

Charges on a body are located very close together such a system of charges said to be continuous distribution of charges

Linear Charge Density :

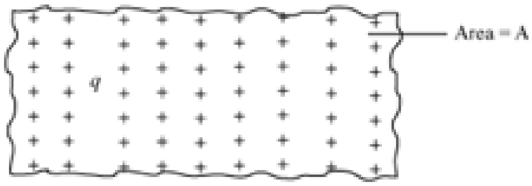
When charge is distributed along a line, the charge distribution is called linear charge distribution and charge present per unit length is known as linear charge density.



$\lambda = \frac{q}{L}$ Where, λ - Linear charge density, q - Total charge distributed along a line of length L.

Unit :- Cm^{-1}

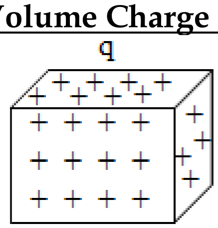
Surface Charge Density



$\sigma = \frac{q}{A}$ Where, σ - Surface charge density, q - Charge distributed on area A,

Unit-Cm^{-2}

Volume Charge Density



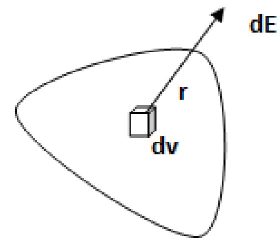
volume (V)

$\rho = \frac{q}{V}$

Where, ρ - Volume charge density, V -Volume of the conductor, q - Charge on conductor

Unit:- $C\text{-}m^{-3}$

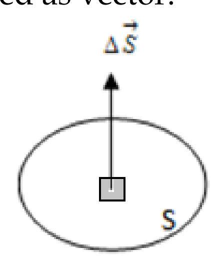
Electric Field Due To Continuous Charge Distribution



Electric field at point A due to element carrying charge Δq is $\Delta \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{r}$ …..(1)

where r is the distance of element under consideration from point A and **r**ˆis the unit vector in the direction from charge element towards point A.

- Total electric field at point A due to all such charge an element in charge distribution is
  $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q}{r_i^2}\hat{r}$ …………………(2)
  where index i refers to the i[th] charge element in the entire charge distribution.
- Since the charge is distributed continously over some region , the sum becomes integral. Hence total field at A within the limit Δq→0 is,
  $\vec{E}_i = \lim_{q_0 \to 0}\frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q}{r_i^2}\hat{r}_i = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$ ……(3)
  and integration is done over the entire charge distribution
  Area Vector: - When a physical quantity passing through any area affected by the orientation of area, then that case area can be treated as vector.



- **Ex.** Amount air passing through an opening of loop , when the opening is held in different orientation with the direction of wind .i.e, zero when plane of opening parallel and maximum when plane is perpendicular.

✓ When area is represented as vector, the length of a vector represents of magnitude and the direction is perpendicular to surface.

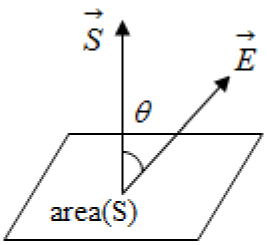
Electric Flux(ϕ_E)

The electric flux, through a surface, held inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

Mathematically:*-It is equal to dot product of electric field and area vector.*

$$\phi = \vec{E}.\vec{S}$$
$$\phi = ES\cos\theta$$

Where θ is angle between field vector $\vec{E}$ and area vector $\vec{S}$.

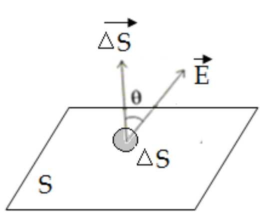


Special cases

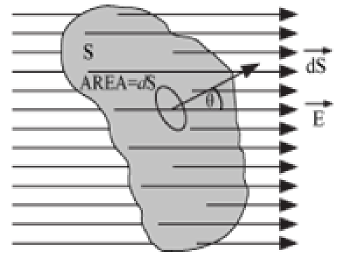
- E is parallel to the area vector (θ = 0) but E is perpendicular to the surface area.
  ϕ = E S cos0= E S (maximum flux)
- If θ = π i. e., electric field vector is in the direction opposite to area vector then
  $\phi = -ES$
- If electric field and area vector are perpendicular to each other then θ = π/2 and ϕ = 0
- if 90 <θ ≤ 180 : Flux is negative
- Flux is positive if θ< 90.

✓ Electric flux is a scalar quantity and it can be added using rules of scalar addition.

✓ SI unit : $Nm^2 C^{-1}$



Electirc flux through an irregular surface:



For calculating total flux through any given surface , divide the surface into small area elements. Calculate the flux at each area element and add them up.

- Thus total flux ϕ through a surface S is
  $$\phi = \sum \vec{E}.\Delta\vec{S}$$
- If we take the limit ΔS→0 and the summation is written as integral
  $$\phi = \int \vec{E}.\vec{ds}$$
  Electric flux through a closd surface is expressed as $\phi = \oint \vec{E}.\vec{ds}$

Gauss Theorem :

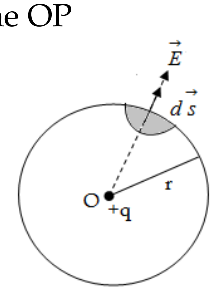
The total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times of the magnitude of thenet charge enclosed by the surface .i.e, $\phi = \frac{q}{\varepsilon_0}$ or

$$\phi = \sum \vec{E}.\Delta\vec{S} = \frac{q}{\varepsilon_0}$$

However, ∴Gauss theorem may be expressed as, $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$

Proof: Consider that a point electric charge q is situated at the centre of a sphere of radius 'r'.

We have, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ Where, $\hat{r}$ is unit vector along the line OP



The electric flux through area element $d\vec{s}$ is given by,

$$d\phi = \vec{E}.d\vec{s}$$
$$= EdsCos\theta$$
$$= EdsCos0^0$$
$$= Eds$$
$$d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$$

Therefore, electric flux through the closed surface of the sphere, $\phi = \oint d\phi = \oint \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$

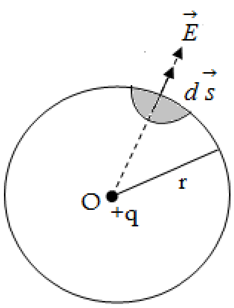
$$\phi = \oint d\phi = \oint \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\times 4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_0}$$

It proves the Gauss theorem in electrostatics.

APPLICATIONS OF GAUSS LAW:
Derivation Of Coulomb's Law



Consider electric field of a single isolated positive charge of magnitude q as shown in the figure. Field of a positive charge is in radially outward direction everywhere and magnitude of electric field intensity is same for all points at a distance r from the charge.

Let os assume Gaussian surface to be a sphere of radius r enclosing the charge q at its centre.

From Gauss's law, $\phi = \sum \vec{E}.\Delta\vec{S} = \frac{q}{\varepsilon_0} \ldots$(i)

since **E** is constant at all points on the surface therefore, $\phi = EA$

$\phi = E(4\pi r^2)$ ……………………………………(ii), surface area of the sphere is $A=4\pi r^2$

from (i) and (ii) $\frac{q}{\varepsilon_0} = E(4\pi r^2)$ *hence*

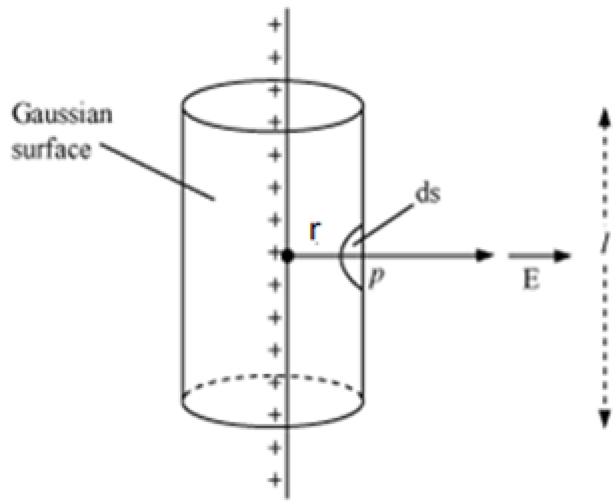
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Now force acting on point charge q_0 at distance r from point charge q is $F=q_0E$

$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$,This is nothing but the mathematical statement of Coulumb's law.

Electric Field Due To A Line Charge

Consider a thin infinitely long straight line charge of linear charge density λ.
Let P be the point at a distance 'r' from the line.
To find electric field at point P, draw a cylindrical surface of radius '*r*' and length *l*.



If *E* is the magnitude of electric field at point P, then electric flux through the Gaussian surface is given by,

ϕ = *E* × Area of the curved surface of a cylinder of radius *r*and length *l*

Because electric lines of force are parallel to end faces (circular caps) of the cylinder, there is no component of field along the normal to the end faces.

$\phi = E \times 2\pi rl \ldots$ (i)

According to Gauss theorem, we have $\phi = \frac{q}{\varepsilon_0}$

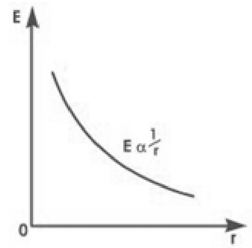
$$\phi = \frac{\lambda l}{\varepsilon_0} \quad \because \lambda = \frac{q}{l}$$

From equations (i) and (ii),

we obtain $E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$

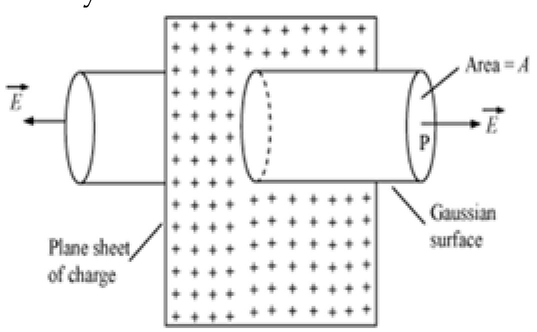
$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{n}$, Where $\hat{n}$ unit vector actiong outwards in direction due to positive charged body



ELECTRIC FIELD DUE TO AN INFINITE PLANE SHEET OF CHARGE

Consider an infinite thin plane sheet of positive charge having a uniform surface charge density σ of the sheet.



Let P be the point at a distance 'r' from the sheet at which electric field is required.
Draw a Gaussian cylinder of area of cross-section A through point P.
The electric flux crossing through the Gaussian surface is given by, $\phi = E \times$ Area of the circular caps of the cylinderSince electric lines of force are parallel to the curved surface of the cylinder, the flux due to electric field of the plane sheet of charge passes only through the two circular caps of the cylinder.

$\phi = E \times 2A$ … (i)

According to Gauss theorem, we have

$$\phi = \frac{q}{\varepsilon_0}$$

Here, the charge enclosed by the Gaussian surface, $q = \sigma A$

$$\phi = \frac{\sigma A}{\varepsilon_0} \text{-----------------------------(ii)}$$

From equations (i) and (ii), we obtain

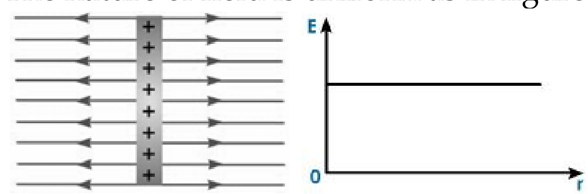
$$E(2A) = \frac{\sigma A}{2\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

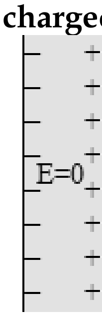
$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$, $\hat{n}$ is unit vector directing outward direction due to positive charge.

$\vec{E} = -\frac{\sigma}{2\varepsilon_0}\hat{n}$ (for a source of negative charge)

- The strength of electric field independent of positon from the charges sheet.
- The nature of field is uniform as in figure.



Eelctric Field due to A thick sheet of charged conductor

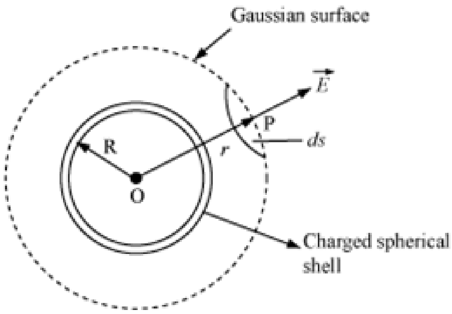


If we follow above case as for thin sheet we will have the charge enclosed by Gaussian surface is
$q = 2\sigma A$ due both surface
so, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

Electric field due to a uniformly charged thin s pherical shell

Case-1(when point p lies outside the spherical shell ($r > R$)

To calculate electric field at the point P at a distance r ($r > R$) from its centre, draw the Gaussian surface through point P so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O.



Let $\vec{E}$ be the electric field at point P. Then, the electric flux through area element $d\vec{s}$ is given by, $d\phi = \vec{E}.d\vec{s}$

Since $d\vec{s}$ is also along normal to the surface, $d\phi = Eds$

∴ Total electric flux through the Gaussian surface is given by, $\phi = \oint Eds = E\oint ds$

$= E(4\pi r^2)$ ---------------------(i)

Since the charge enclosed by the Gaussian surface is q, according to Gauss theorem,

$\phi = \frac{q}{\varepsilon_0}$ ------------------------(ii) From equations (i) and (ii), we obtain $E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$...........(iii)

If σ surface charge density. Then $\sigma = \frac{q}{4\pi R^2}$ or

$\sigma 4\pi R^2 = q$

Equation(iii) also written as $E = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{r^2}$ or

$E = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}$

Case:-2 When Point P Lies Inside The Spherical Shell (r<R)

In such a case, the Gaussian surface encloses no charge.

According to Gauss law, $E \times 4\pi r^2 = 0$ i.e., $E = 0$

Case-3(When The Point Lies On The Surface r=R)

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

$\because \sigma 4\pi R^2 = q$ or $E = \frac{\sigma}{\varepsilon_0}$